# Aperiodic Tiling of Diffusers Using a Single Asymmetric Base Shape

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## Abstract

If diffusers are placed in a periodic arrangement, the scattered energy will be concentrated into diffraction lobes, and consequently the scattering will not be uniform in all directions. Furthermore, the low frequency response can be limited by periodicity effects. Consequently, this paper will present a method for creating and arranging diffusers to cover a large area in an aperiodic manner to overcome these problems. The method is applicable to curved surfaces, including hemispherical scatterers. It uses a single asymmetric base shape for ease of manufacture and installation. The effectiveness of the technique will be demonstrated through Boundary Element Modeling.

## 1. Introduction

Acoustic diffusers are often arranged in arrays. In that situation, the performance of the diffuser depends both on the scattering ability of one of the diffusers, and on the diffuser arrangement within the array. While pleasing to the eye, periodic arrays are often undesirable, because they lead to concentration of scattered energy in specific diffraction directions, which are determined by the size of the repeat unit. Figure 1 shows the scattering from a single semicylinder, and 12 semicylinders arranged in a periodic array. While the single semicylinder does produce uniform dispersion, on its own it cannot cover a sufficient wall area to be useful. Once the semicylinders are placed in an array to cover the wall, it is the array characteristics, typified by the grating lobes that dominate the scattering.

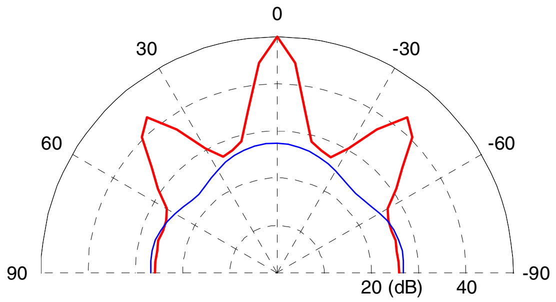



*Figure 1. Scattering from ——— a single semicylinder, and ·········· 12 of the same semicylinders.*

Early reflection phase gratings utilized periodicity to provide lobes of equal energy in these specific diffraction directions to mimic uniform diffusion. As Figure 1 illustrates, when the number of lobes is small, the spatial dispersion is not optimal regardless of whether the lobes have equal energy. This becomes problematic when low order phase gratings are used, as is most common, as this produces sparse grating lobes across the most important frequency ranges for acoustic quality.

One solution to these grating lobes is to remove the periodicity completely by using a very large surface. Unfortunately, this is likely to be an expensive solution. For this reason, a modulation scheme, which achieves large area coverage from a few base shapes, is desirable. Early work in this area by Angus concentrated on Schroeder diffusers [1-2]. Two different Schroeder diffusers are used, say one being an N=5 QRD and the other an N=7 QRD. These diffusers are then arranged, with the order of the diffusers being determined by a pseudo-random number sequence with good aperiodic autocorrelation properties to minimize periodicity effects. This technique is most successful in producing uniform diffusion if the second base shape is the inverse of the first – i.e. the base shapes produce scattering which is 180  out of phase [3]. However, it is more efficient for manufacture if a single asymmetric base shape can be used, as might be achieved using a primitive root sequence. Furthermore, inverse diffusers are only easily obtained for phase gratings. Nowadays, diffuser design has moved on from phase gratings to other shapes, such as curves, which complement modern architecture. Consequently, a technique for dealing with periodicity for other surface shapes is needed.

Consider a single asymmetrical diffuser base shape as shown in Figure 2. If this base shape was arranged in a periodic fashion, grating lobes will arise. If, however, some of the periods are rotated, then the periodicity can be reduced. In the case shown in Figure 2, the repeat distance has been doubled by this modulation. In general this will improve the diffusion.

A method has been devised to optimize a single asymmetric curved base shape, which can be modulated by rotation and yet seamlessly tile with adjacent units in any orientation [4]. The necessary conditions for seamless tiling and high acoustic performance will be given. Surface constructions, which enable the designer to choose the final array pattern, will be presented. 3D boundary element modeling will demonstrate the performance of these new surfaces, where it will be compared to other common diffuser types. The optimized base shape and modulation offer a highly diffusing surface.

## 2. Base Shape Optimization

The asymmetric base shape is optimized using methods previously described [5]. The technique is to task a computer to search for the diffuser which produces the best dispersion through an iterative (trial and error) procedure. This process allows arbitrary shapes to be optimized for their acoustic performance, and also to meet the visual requirements of a space.

In this paper, the interest is to produce a diffuser which can seamlessly tile in any orientation. For this reason, the diffuser must be optimized within a modulated array. If only one element or a periodic arrangement is considered during optimization, then the element may not be sufficiently asymmetrical to break up the grating lobes when put in a modulated arrangement. The need for seamless tiling in any orientation places additional constraints on the shapes that can be produced. The asymmetric base shape has to be made square, with symmetrical and identical sides with a perimeter gradient of zero. This allows the base shape to be seamlessly attached to an adjacent unit in any orientation. If the surface depth is z and is a function of the coordinates across the width x and length y of the diffuser, z = F(x,y), then the requirements can be mathematically expressed as:

$$
\begin{aligned}
&z(0,y) = z(L,y) \\
&z(x,0) = z(x,L) \\
&z(x,0) = z(L-x,0) \qquad (1) \\
&z(x,0) = z(0,x) \\
&\frac{\partial z}{\partial x} = 0 \quad x = 0 \vee L \\
&\frac{\partial z}{\partial y} = 0 \quad y = 0 \vee L
\end{aligned}
$$

where L is the width (or length) of the diffuser.

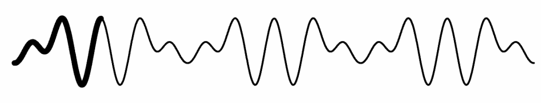


*Figure 2. Tiling a single plane diffuser. The base shape is shown in bold. (After Cox & D'Antonio [3])*

The function F() representing the surface shape can take a variety of forms. In the initial work on optimizing curved surfaces [5] this was a truncated Fourier series. Unfortunately, it proves to be difficult to achieve the necessary perimeter conditions set out in Equation (1). It is possible to construct a harmonic series not based on sinusoidal basis functions and solve these problems. Alternatively, frequency and amplitude modulation processes can be used to generate many different shapes. Probably the most useful representation is achieved using a bicubic spline algorithm [6], which is the two dimensional equivalent of the more familiar cubic spline algorithm used to carry out curve fitting in one dimension.

In Figure 3, an asymmetric base shape at 0, 90, 180, and 270 degrees of rotation is shown, to illustrate how there are no discontinuities between adjacent oriented base shapes. Also shown is a seamless area consisting of a 4 x 4 array of randomly oriented base shapes. While the conditions set out above ensure that the elements tile in

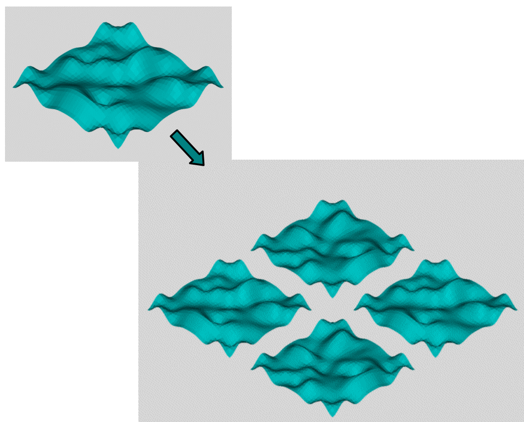
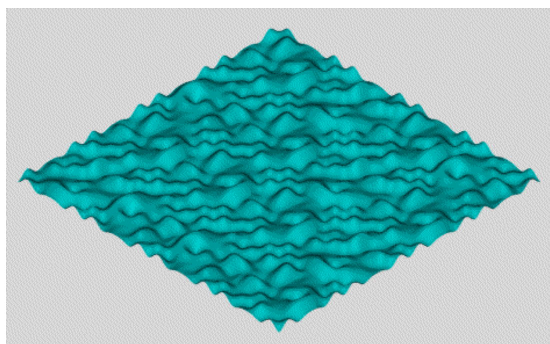


*Figure 3. Seamless tiling of a single base shape (above), a 4x4 array (bellow)*



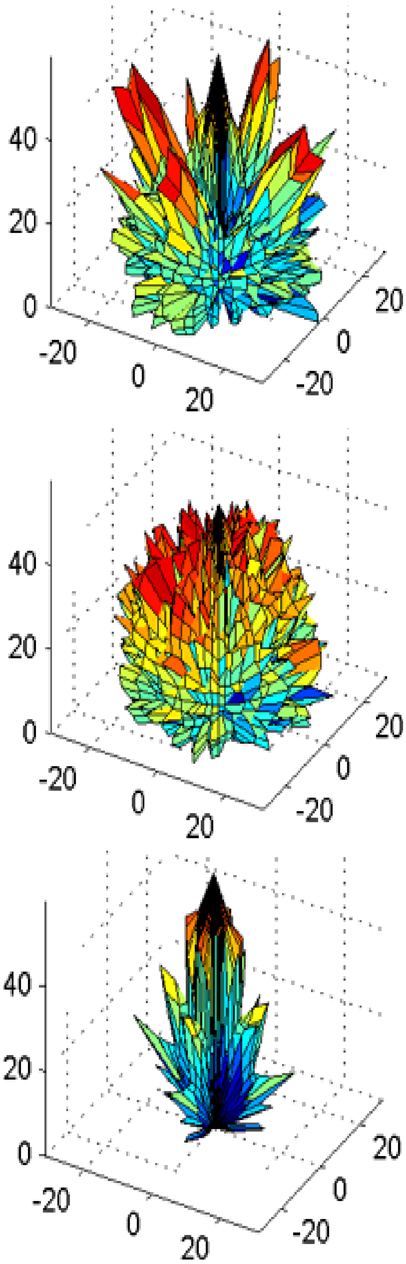



*Figure 4. 2 kHz polar responses: top, periodic array; middle, modulated array; bottom, plane surface*

any orientation, the tiling may become apparent if care is not taken.

## 3. Performance

The performance can be evaluated using polar responses and diffusion coefficients [7]. In Figure 4 the polar responses at 2 kHz are shown at the top for a periodic array. Modulating the asymmetric base shape into an array minimizes the focusing of scattered energy into the diffraction directions visible in the top illustration. This is shown in the middle polar response. The polar response for a plane surface is shown for comparison at the bottom.

The diffusion coefficient spectra shown in Figure 5 summarize the uniformity of diffusion versus frequency. (As is common with the autocorrelation coefficient, for hemispherical scatterers the coefficient is numerically rather small). It shows the scattering from the modulated array is much greater than the periodic arcs. At higher frequencies, beyond the range predicted, it would be anticipated that the periodic and modulated arrays would give similar performance. This would happen when there is a sufficiently large number of grating lobes in the periodic case.

### 3.1. Bass response

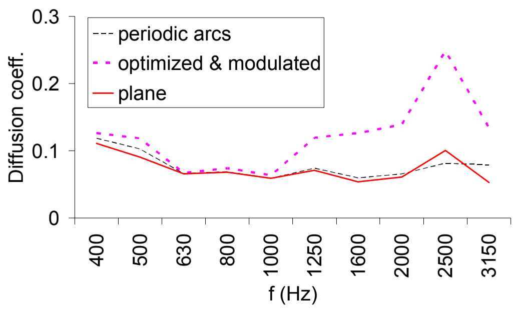



*Figure 5. Diffusion coefficient for periodic arcs, modulated diffuser and plane surface*

One aspect of modulation not yet discussed is the effect this has on the bass response of diffusers. The low frequency response of a periodic diffuser is limited by two factors: (i) the repeat distance and (ii) the diffuser depth. If a diffuser is narrow (has a small repeat distance), then only one scattering lobe is generated in the direction defined by Snell's law. The only effect that the depth of the diffuser can have is to change the level of the lobe, it can not provide significant sound scattering in other directions.

For significant scattering, the repeat distance must be long enough for the surface to generate additional grating lobes. Consider scattering in one plane only for simplicity. The directions of the lobes in a periodic array can be calculated by simple geometric considerations [8]:

$$\sin(\theta) = m\lambda / W - \sin(\psi) \quad m = 0, \pm 1, \pm 2, \ldots \tag{2}$$

where  is the angle of $m^{th}$ lobe,  the wavelength, W the repeat distance and  the angle of incidence.

Consequently, for normal incident sound, W≥ to generate scattering, and preferably W>> . As modulation naturally extends the repeat distance, it enables the diffuser response to be limited by depth and not repeat distance. This is why the periodic arc performance in Figure 5 is no better than the plane array; the performance was limited by repeat distance.

## 4. Visual aesthetics

Figure 6 shows the application of a similar type of diffuser in a worship space. The diffuser shown only has tiling ability in one direction, and the architect has not exploited the seamless capability, placing spaces between the elements to allow for lighting.

*Figure 6. Worship space application*

The architect will usually decide on the visual desirability of acoustic treatment. Unfortunately, from an acoustic point of view, a periodic look is often favored, whereas a modulation forms a random appearance. It seems that a periodic object enables the eye to more easily decode the design. Using this asymmetrical base shape modulation, with the base shape formed by optimization, gives designers more control over the appearance. It can be made to look random or periodic as desired, but the designers have to remember that short repeat distances will result in worse dispersion. The best solution might be a periodic one, with a sufficiently long repeat distance, but that is a matter of taste. To take a final example, Figure 7 shows a modulated array where a distinct pattern has been formed which follows the well known log cabin quilt pattern. As with the log cabin quilts, rearrangement of these blocks allows a vast variation in the patterns generated. Again only a single base shape is being used.

## 5. Conclusions

A method for designing curved diffusers to minimize the effects of periodicity has been outlined. Periodicity can limit the dispersion generated by diffusers by creating energy concentrated in lobe directions. Periodicity can also limit the low frequency dispersion. By using a single asymmetric base shape, optimized for good acoustical performance, it is possible to increase the repeat distance, and hence improve performance. It also creates a new aesthetic for architects and designers.

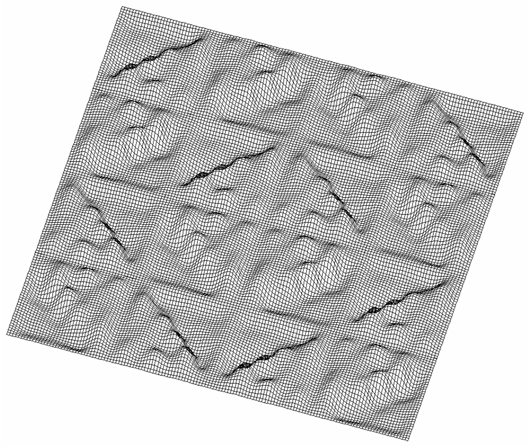


*Figure 7. A modulated diffuser based on the log cabin quilt pattern (After Cox and D'Antonio [3]).*

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